

Development of new models for predicting the oil recovery of sandstone reservoirs

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Abstract

In this paper, a new correlation was developed (139 data sets) in this study to predict the oil recovery for sandstone solution gas drive reservoirs (SGDR) using NLMR with a coefficient of determination, R^2 of 0.90 (compared to 0.87 for Gulstad correlation). An artificial neural network (ANN) model was developed for this reservoir giving an R^2 of 0.92. A correlation was developed (111 data sets) in this study to predict the oil recovery for sandstone water drive reservoirs (SWDR) using NLMR with a coefficient of determination, R^2 of 0.93 (compared to 0.91 for American Petroleum Institute, API correlation). The developed ANN model for this reservoir gives an R^2 of 0.94.

ARTICLE HISTORY

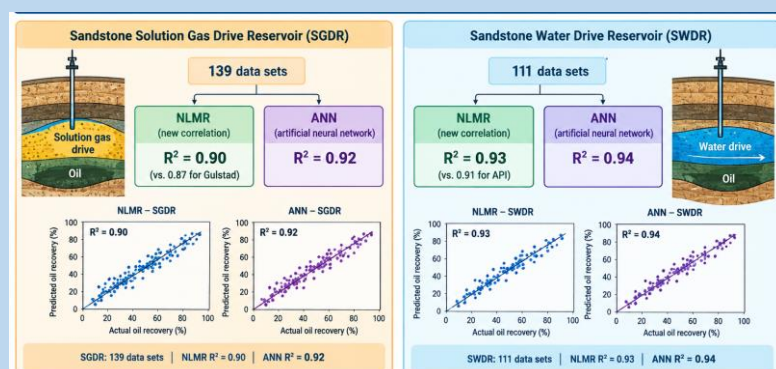
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1. Introduction

The oil recovery factor refers to the percentage of the original oil in place that can be commercially produced. Depending on this parameter, we can determine whether the production from a certain oil reservoir is feasible or not. Also, all the production facilities are designed based on this parameter and based on the allowable oil production rate. There are many methods for predicting the oil recovery factor which differ in accuracy from one method to another. This paper aims to review and evaluate the published oil recovery correlations and develop more accurate models compared to the published ones.

2. Literature Review

In a statistical study of Craze and Buckley's water-drive recovery data, Guthrie and Greenberger, using multiple correlation analysis methods, found the following correlation between water-drive recovery and five variables that affect recovery in sandstone reservoirs (Guthrie and Greenberger 1955) (Ahmed 2018)

$$R = 0.114 + 0.272 \log(k) + 0.256 \log(S_w) - 0.136 \log(\mu_o) - 1.538(\phi) - 0.00035(h) \quad (1)$$

Based on 73 sandstone reservoirs, Guthrie and Greenberger (1963) developed an empirical expression that correlates the recovery factor with Permeability, k , md Porosity, ϕ , as a fraction Oil viscosity, μ_o , cp Connate water saturation, S_{wc} , as a fraction Reservoir thickness, h , ft The empirical equation for predicting the water flood ultimate recovery factor (RF) takes the form (Guthrie and Greenberger 1955) (Ahmed 2018)

$$RF = 0.2719 \log K + 0.25569 S_{wc} - 0.1355 \log \mu_o - 1.538 \phi - 0.0008488 h + 0.11403 \quad (2)$$

Arps (1967) statistically correlated the recovery factor as a function of the initial pressure p_i and pressure at depletion p_a by the following

relationship (Onolemhemen, Isehunwa, and Salufu 2016) (Ahmed 2018):

$$RF = 54.898 \left[\frac{\phi(1 - S_{wi})}{B_{oi}} \right]^{0.0422} \left[\frac{\mu_{wi} k}{\mu_{oi}} \right]^{0.077} (S_w)^{-0.1903} \left(\frac{P_1}{P_a} \right)^{-0.2159} \quad (3)$$

The American Petroleum Institute calculated the oil recovery factor from a water-drive reservoir using eight factors in sandstone reservoirs: porosity, water saturation, initial oil formation volume factor, permeability, water and oil viscosities at the initial conditions, and initial and abandonment pressures (Gulstad 1995).

$$R = 4259 \frac{\phi[1 - S_w]^{1.0422}}{B_{oi}} \frac{k \mu_{wi}^{0.0770}}{\mu_{oi}} [S_w]^{-0.1903} \left[\frac{p_i}{p_a} \right]^{-0.2159} \quad (4)$$

The American Petroleum Institute developed also, another correlation for sandstone solution gas drive reservoirs as follows:

$$R = 3244 \left[\frac{\phi(1 - S_w)}{B_{ob}} \right]^{1.1611} \left[\frac{K}{\mu_{ob}} \right]^{0.0979} [S_w]^{0.3722} \left[\frac{P_b}{P_a} \right]^{0.1741} \quad (5)$$

The correlations show an extremely strong dependence of recoverable oil on the original oil-in-place. However, other reservoir and fluid parameters were included in the correlation since calculations showed a statistically significant increase in the correlation coefficient when using the multiple-factor regressions (Gulstad 1995).

Gulstad conducted a study on "The Determination of Hydrocarbon Reservoir Recovery Factors by Using Modern Multiple Linear Regression Techniques". During the study, the recovery factor of hydrocarbon reservoirs was observed from the beginning of the well's life to the end, to leverage the benefits of earning during the production stage. Additionally, knowledge gained

from the calculation of the recovery factor allowed for improved programs aimed at developing the primary recovery factor. The equations for the recovery factor, determined using multiple linear regression models, are as follows (Gulstad 1995):

The equation for sandstone reservoir – solution gas drive:

$$\begin{aligned} R &= -264.59 + 0.34(OOIP) + 29.37 \ln(R_{SI}) \\ &- 0.06(\lambda_o) + 10.70 \ln(\lambda_o) \\ &- 12.64 \ln(h) \end{aligned} \quad (6)$$

The equation for sandstone reservoir- water drive:

$$\begin{aligned} R &= -274.94 + 0.44(OOIP) - 56.70 \ln(\mu_{oa}) - \\ &119.45 \ln(S_w) + 0.04(P_{cp}) - 4.73(\mu_{oi}) + \\ &4.38(\mu_{oa}) + 0.24(OOIP_{calc}) - \\ &0.88(T) \end{aligned} \quad (7)$$

The equation for carbonate reservoir- solution gas drive:

$$\begin{aligned} R &= -711.29 + 0.30(k) + 46.64 \ln(OIIP) \\ &+ 420.18(\mu_w) + 0.01(P_i) + 2.19(API) \\ &- 190.00 \ln(\mu_w) \\ &+ 79.32(S_w) \end{aligned} \quad (8)$$

The equation of carbonate reservoir- water drive:

$$\begin{aligned} R &= -177.26 - 57.11(\mu_{bp}) + 0.34(OOIP) \\ &+ 33.90(\mu_{oi}) + 0.17(h) \\ &- 86.83 \ln(P_i) + 283.27 \ln(B_{oa}) \\ &+ 75.19 \ln(\mu_{bp}) \\ &+ 175.89 \ln(T) \end{aligned} \quad (9)$$

Gulstad concluded that the four equations for the recovery factor depended on the original oil-in-place and a combination of reservoir rock properties. The best model was found to be the equations for carbonate reservoirs with a water drive mechanism. The values of coefficient of determination for carbonate reservoirs were found to be greater than those for sandstone reservoirs. Additionally, the study found that the heterogeneity of a reservoir plays an important

role in characterizing the hydrocarbon reservoirs and estimating the recovery (Gulstad 1995).

Noureldien and El-Banbi studied estimating oil recovery factors using artificial intelligence or calculating reserves wasn't an easy task it needed many conditions and parameters. The first step of the study was to standardize and normalize the input and output parameters. Then, they classified the parameters into two main categories: quantitative parameters and qualitative parameters. Quantitative parameters were (permeability, porosity... etc.) but these data were not normally represented in digits so they used actual field data. These data included parameters such as (facilities, start of production, and number of wells.) then they built two artificial neural networks (ANNs): the first had simple and basic data and the second had advanced data with reasonable accuracy. They concluded that new ANN models were developed for estimating the oil recovery factor but they did not publish the ANN model (Noureldien and El-Banbi 2015).

Olugbenga Olamigoke et al. Studied primary recovery factor correlations for thin oil rims with large gas caps. They found a correlation between the primary recovery factor and a large gas cap. This correlation estimated depended on three dimensions. The correlation for calculating the oil recovery factor in conventional development was validated using real production data from an oil rim field. On the other hand, the models for concurrent development were validated using reservoir simulation results that were matched with historical data. The oil recovery factor . estimated from the correlation of this study was highly accurate, as it matched well with the field data. It was worth noting that the developed correlation has been tested and validated for reservoirs with oil columns less than 100 ft underlying large gas caps. Therefore, they could be applied to such

reservoirs with a good degree of confidence. The correlation that developed for estimating the ultimate oil recovery on the vertical well was (Olamigoke and Isehunwa 2019):

$$N_{pw} = \frac{\phi(1 - S_{wc} - S_{or})k_h h_o^{2.5} NTG^{1.5}}{\mu_{oi} B_{oi} * 10^6} \quad (10)$$

These response surface models, also known as correlations, were created by fitting non-linear regression models to oil recovery estimates obtained from reservoir simulation runs. These estimates were determined using Box-Behnken experimental design for both conventional development and concurrent development at gas cap production rates of 5% and 10% of free gas initially in place per annum. Then they get an equation for recovery factor which applied to both conventional and concurrent oil and gas development using horizontal wells:

$$RF = 4.8 + 0.19h_o * 3.1 * 10^{-3}k_h - 0.5m + 0.4r_{eD} - 1.9\mu_o + 1.8\left(\frac{k_v}{k_h}\right) + 0.1\theta + 4.87q_{gi} + 3 * 10^{-5}h_o k_h + 0.01mh_o + 0.01h_o r_{eD} + 5.5mk_h - 0.06h_o\mu_o - 5.5 * 10^{-3}h_o q_{gi} - 3 * 10^{-4}k_h r_{eD} - 1.3 * 10^{-4}k_h q_{gi} \quad (11)$$

$$R_s = \gamma_g API \left[\left(\frac{P + 218.2}{172.4} \right) \left(\frac{T}{API \gamma_g} \right)^{-0.5592} \right]^{\frac{1}{0.5852}} \quad (12)$$

The final result showed that the dominant factors affected by oil rim and gas cap production were oil rim thickness, gas cap size, oil viscosity, horizontal permeability, aquifer radius, gas cap offtake, vertical anisotropy, and reservoir dip.

Mahmoud et al. estimated the oil recovery factor for water drive sandy reservoirs through

applications of artificial intelligence accuracy of these applications depended on existing data and reservoir age. They

used 10 parameters that were: the effective reservoir thickness, original oil in place, reservoir pressure, the reservoir area, porosity, Lorenz coefficient, effective permeability, API gravity, oil viscosity, and initial water saturation. The researchers used AI models to develop an empirical correlation for recovery factor estimation based on data collected from a large number of water-drive sandstone reservoirs. The AI models were trained using data from 130 reservoirs and then tested using data from 38 other reservoirs. The results showed that the ANNs-based correlation developed by the researchers outperformed the currently available empirical equations for recovery factor estimation in terms of all the measures of error evaluation used in the study. The ANNs-based correlation also had a higher coefficient of determination (0.94) compared to the Gulstad correlation, which was considered to be one of the most accurate correlations currently available (with a coefficient of determination of only 0.55) (Mahmoud et al. 2019).

3. Results and Discussion

3.1 Data preparation and description for sandstone solution gas drive reservoirs (SSGDR)

139 datasets are collected from the literature. The data description is presented in Table 1. The data represents the oil recovery as a function of the original oil in place, initial solution GOR, oil mobility at the bubble point pressure, and net pay thickness. As presented in this table, the collected data covers oil recovery values ranging from 16 to 908 STB, the original oil-in-place values ranging from 189 to 2146 STB, initial solution GOR values ranging from 44 to 3000 SCF/STB, oil mobility values range from 0.11 to 6230.769

and net pay thickness values range from 4.5 to 1100 ft.

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Table 1: Data description for predicting oil recovery of sandstone solution gas drive reservoirs.

	N	Rsi	k/uob	H	REC
Mean	828.95	577.518	163.705	63.319	187.324
Standard Error	32.72	40.545	48.179	10.428	12.645
Median	735	480	33.333	25	138
SD	385.766	478.024	568.025	122.947	149.079
Sample Variance	148815.4	228507	322652.7	15115.94	22224.6
Kurtosis	1.318	8.297	95.77	37.571	5.437
Skewness	1.195	2.41	9.143	5.294	2.101
Range	1957	2956	6230.659	1095.5	892
Minimum	189	44	0.11	4.5	16
Maximum	2146	3000	6230.769	1100	908
Count	139	139	139	139	139

Before developing the correlation, we should check the relation between the oil recovery and the input parameters. **Figure 1** shows the effect of the input parameters on the oil recovery for the sandstone solution gas drive. As depicted in this figure, the oil recovery is directly proportional to the original oil in place, oil mobility, and net pay thickness. On the other hand, the oil recovery is inversely proportional to the initial solution GOR. As shown also, in this figure, the oil recovery is affected by the original oil in place with coefficient correlation, $R = 78\%$, initial solution GOR with $R = -27\%$, oil mobility with $R = 15\%$, and net pay thickness with $R = 18.6\%$.

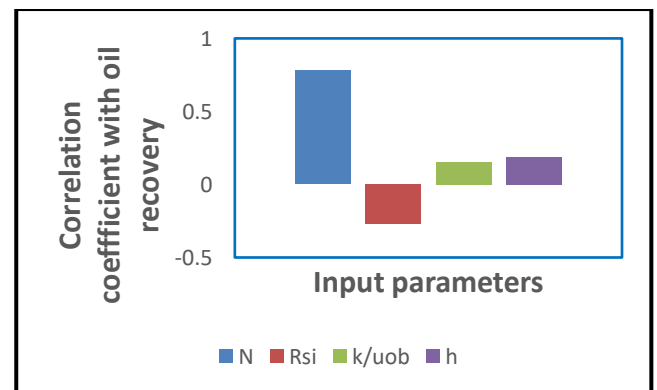


Figure 1: Effect of input parameters on oil recovery for SSGDR

3.2 Development of the new correlation using nonlinear multiple regression (NLMR) for SSGDR:

A correlation was developed in this study to predict the oil recovery. The correlation was developed to predict the oil recovery as a function of the original oil in place, oil mobility, and net pay thickness, solution GOR. To develop this correlation 139 datasets are

collected from the literature. The developed oil recovery correlation for SSGDR can be expressed as:

$$R = aN^b(R_{si} + c)^d(k/\mu_o)^e h^f \quad (13)$$

Where:

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>e</i>	<i>f</i>	<i>g</i>
0.0377	1.696	-28.618	0.167	1798.7	-0.4877	-0.0672

Figure 2 shows the calculated versus actual recovery factor. As shown in **Figure 2** there is an accepted correlation between them where the developed correlation achieves a coefficient of determination, R^2 of 0.90.

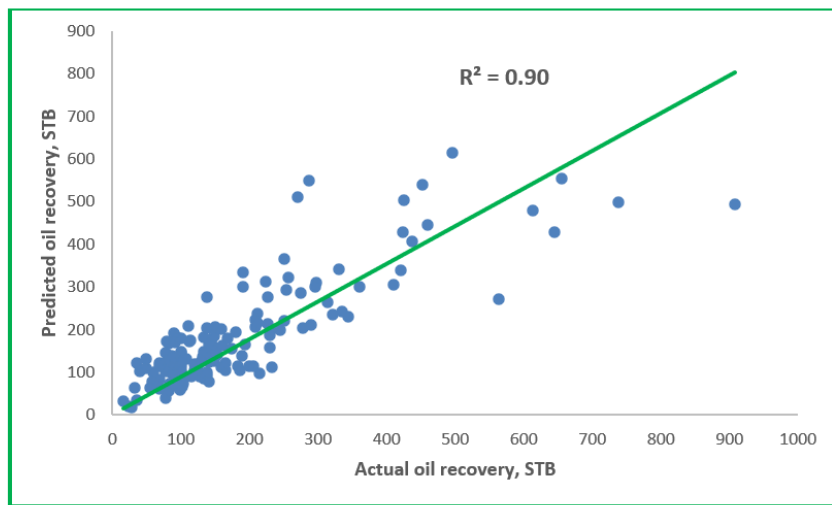


Figure 2: Cross plot for oil recovery of SSGDR (after this study)

We investigated the accuracy of the most common published correlation to predict oil recovery compared with the model developed in this study. The correlation for oil recovery of this study achieved the highest coefficient of determination of 0.90 as shown in **Figure 2**. The oil recovery Gulstad correlation gives a coefficient of determination of 0.87 as depicted in **Figure 3**.

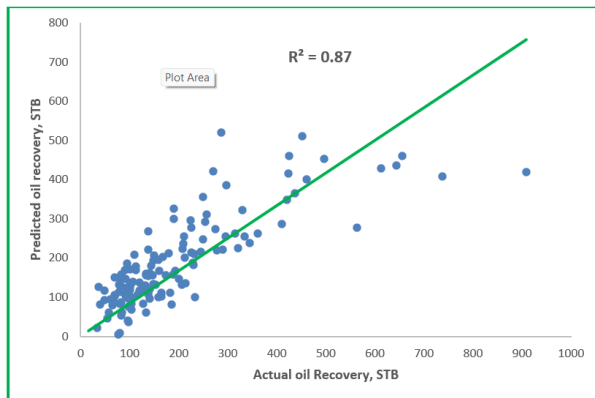


Figure 3: Cross plot of the oil recovery correlation (after Gulstad)

3.3. Data preparation and description for sandstone water drive reservoirs (SWDR)

111 datasets are collected from the literature. The data description is presented in **Table 2**. The data represents the oil recovery as a function of the original oil in place, porosity, water saturation, permeability, pressure, net pay thickness, and API. As presented in this table, the collected data covers oil recovery values ranging from 30 to 1470 STB, the original oil-in-place values ranging from 242 to 2062 STB, porosity values ranging from 0.11 to 0.35 %, water saturation values range from 0.07 to 0.53, permeability values range from 7 to 5000 D, pressure values range from 1 to 14.94943, net pay thickness values range from 6 to 600 ft, and API values range from 14 to 60.

Table 2: Data description for predicting oil recovery of SWDR.

	OOIP	Poro	Sw	K	Pi/Pa	H	API	REC
Mean	1180.243	0.265856	0.266739	815.473	2.70424	59.43964	33.65135	604.1261
Standard Error	32.96195	0.005597	0.009583	85.69972	0.29938	8.177261	0.737456	30.62193
Median	1196	0.288	0.26	440	1.400585	31	35	600
Mode	1007	0.3	0.3	1000	1	16	35	750
Standard Deviation	347.2757	0.058964	0.100966	902.9026	3.154169	86.15279	7.769578	322.6221
Sample Variance	120600.4	0.003477	0.010194	815233.2	9.948779	7422.303	60.36634	104085
Kurtosis	-0.25654	-0.39058	-0.27357	4.548142	6.732518	15.71766	0.450995	-0.32433
Skewness	-0.099	-0.78092	0.274574	1.979075	2.695504	3.531374	0.208689	0.375767
Range	1820	0.24	0.46	4993	13.94943	594	46	1440
Minimum	242	0.11	0.07	7	1	6	14	30

Maximum	2062	0.35	0.53	5000	14.94943	600	60	1470
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Before developing the correlation, we should check the relation between the oil recovery and the input parameters. **Figure 4** shows the effect of the input parameters on the oil recovery for the sandstone water drive. As depicted in this figure, the oil recovery is directly proportional to the original oil in place, porosity, and permeability, and net pay thickness. On the other hand, the oil recovery is inversely proportional to the water saturation, pressure, and API.

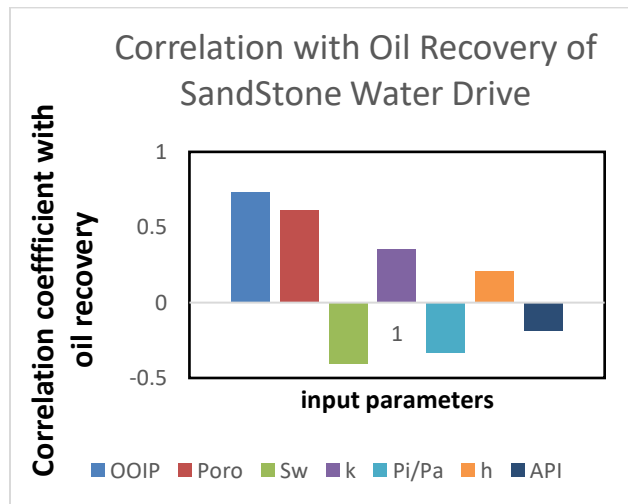


Figure 4: Effect of input parameters on oil recovery for SWDR

As shown also in this figure, the oil recovery is affected by the original oil in place with coefficient correlation, $R = 0.730$, porosity with $R = 0.61$, water saturation with $R = -0.41$, and permeability with $R = 0.36$, pressure with $R = -$

0.33 and net pay thickness with $R = 0.21$ and API with $R = -0.19$

3.4 Development of the new correlation using NLMR for SWDR:

A correlation was developed in this study to predict the oil recovery. The correlation was developed to predict the oil recovery as a function of the original oil in place, porosity, water saturation, permeability, pressure, net pay thickness, and API. To develop this correlation 111 datasets are collected from the literature.

The developed oil recovery correlation of SWDR can be expressed as:

$$RF = a + ooip^b * \phi^c * Sw^d * K^e * pi/pa^f * h^g * API^h \quad (14)$$

$a = 0.485045$, $b = 0.874907$, $c = 0.764819$, $d = 0.331989$, $e = 0.023058$,

$f = 0.102882$, $g = 0.056389$, $h = 0.417031$

Figure 5 shows the calculated versus actual recovery factor. As shown from this figure there is a good correlation between them where the developed correlation achieves a coefficient of determination, R^2 of $= 0.93$.

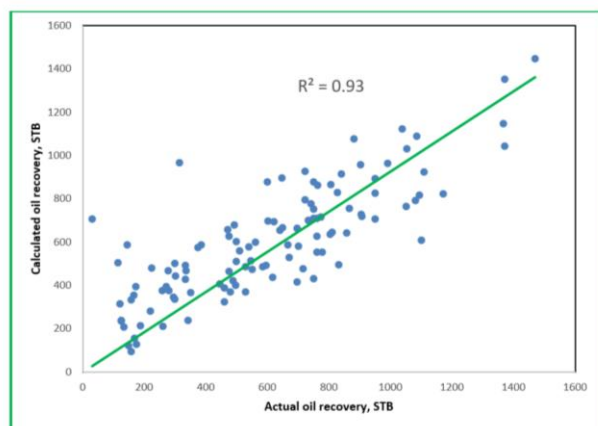


Figure 5: Cross plot of the oil recovery correlation.

We investigated the accuracy of the most common published correlation to predict the oil recovery compared with the models developed in this study. The correlation for oil recovery of this study achieved the highest coefficient of determination of 0.93 as shown in **Figure 5**. The oil recovery API correlation gives a coefficient of determination of 0.91 as depicted in **Figure 6**.

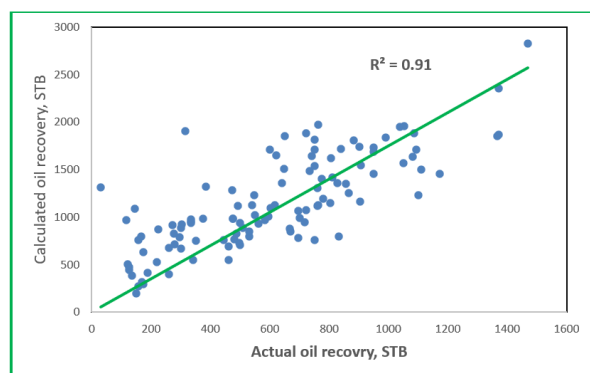


Figure 6: Cross plot for oil recovery (API correlation)

3.5 Development of oil recovery for SSGDR using artificial neural network (ANN)

The collected data sets are divided into three sets before building the ANN model to predict the oil recovery. 70% of the data sets are employed for the process of model training, while the remaining 30% of data sets are used for validation and testing the performance of the model. Besides, all the data are normalized between -1 and 1 to build the ANN model.

In this work, an ANN model is developed to estimate the oil recovery as a function of the original oil in place, oil mobility, and net pay thickness, solution GOR. The model is built based on three layers. The first layer is the input layer, which has 4 neurons for inputs. 9 neurons contribute to the hidden layer, which is the second layer. The output layer, which has one neuron to predict the output parameter, oil recovery, is the third layer we first examined the Tan-sigmoid as a transfer function at different numbers of hidden neurons (5, 6, 7, 8, 9, and 10) as presented in **Table 3**. We found the highest coefficient of determination and the lowest RMSE at $n = 6$. Then we examined the logistic-sigmoid as a transfer function at numbers of neurons of (5, 6, 7, 8, 9, and 10) as presented in **Table 4**. We found that the optimum transfer

function is Log-sigmoid with 9 neurons where the coefficient of determination is 0.92 and MAE is 20.9 %. To reach this fact a Levenberg-Marquardt technique was selected as a training algorithm and the pure-linear was examined to be the output function. **Figure 7** shows the layout of the proposed model and the characteristics of the proposed model are presented in **Table 5**.

Table 3: Oil recovery model accuracy at various numbers of neurons in the hidden layer with a tan-sigmoid transfer function for the SSGDR

	5	6	7	8	9	10
R2	0.51	0.75	0.32	0.38	0.54	0.16
SD	29.93	20.69	34.86	22.61	25.83	67.87
RMSE	46.54	29.99	46.44	42.12	51.63	84.17
RE	-	-	-19.6	-	-	-
AE	33.95	22.66	34.85	26.23	30.05	88.37

Table 4: Oil recovery model accuracy at various numbers of neurons in the hidden layer with log-sigmoid transfer function for the SSGDR

	5	6	7	8	9	10
R2	0.6	0.57	0.27	0.54	0.92	0.39
SD	26.95	27.15	27.31	25.04	18.97	29.42
RMSE	43.45	48.31	58.86	51.36	22.12	59.21
RE	-	-	-	-	-	-
AE	32.53	32.95	32	35.13	20.9	30.26

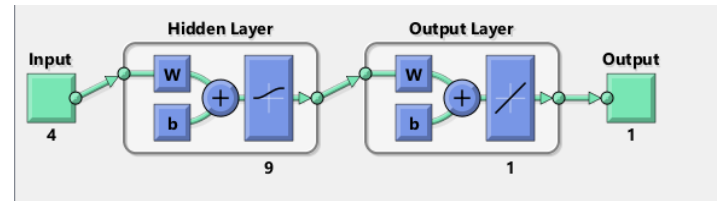


Figure 7: Layout of the proposed ANN oil recovery model for SSDR

Table 5: Characteristics of ANN oil recovery model of the SSDR

Parameter	Value
ANN layers number	3
neurons number in the input layer	4
optimum neuron number in the hidden layer	9
training algorithm for the neural network	Levenberg-Marquardt
transfer function of the hidden layer	Log sigmoid
transfer function of the output layer	pure linear

The developed model for oil recovery using the ANN can be expressed as follows:

First, the input parameters are normalized using the following expressions:

$$N_n = 0.001022 * N - 1.193153 \quad (15)$$

$$R_{s_n} = 0.000677 * R_s - 1.02977 \quad (16)$$

$$\begin{aligned} & (k/\mu)_n \\ & = 0.000321 * \left(\frac{k}{\mu}\right) \\ & - 1.000035 \end{aligned} \quad (17)$$

$$\begin{aligned} & h_n \\ & = 0.001826 * h \\ & - 1.008215 \end{aligned} \quad (18)$$

For $i = 1$ to no of neurons and for $j = 1$ to no of inputs, the inputs for the hidden are calculated from the following expression:

$$S_{i,j} = \sum_{j=1}^N (w_{i,j}x_j) + b_i \quad (19)$$

Where x_j represents the normalized input parameter.

The oil recovery can be calculated by the following function:

$$\begin{aligned} \text{REC} = 446 \left[b_{ho} + \sum_{i=1}^n w_{hoi} \left(\frac{1}{1 + \exp(-S_i)} \right) \right] \\ + 462 \end{aligned} \quad (20)$$

The proposed oil recovery model's coefficients are listed in **Table 6**.

The regression plot for the oil recovery model of SSDR, as indicated in **Figure 8** presents the relation between the calculated and the true oil

recovery for all data sets. The fit is accepted where the coefficient of determination is 0.92

Table 6: Weights and Biases of the Oil Recovery ANN Model for SSGDR

Neuron #	Wi,1	Wi,2	Wi,3	Wi,4	bi	Whi	Bh
1	-3.4489	0.82109	2.2473	-2.4703	4.8571	-0.17252	-0.93786
2	-3.2534	-1.5967	-0.4458	3.1584	3.6523	-0.8374	
3	-1.9765	3.0192	-2.9843	-1.4835	2.2211	1.0455	
4	1.2276	-3.0785	-1.9261	2.9099	-1.3954	0.14924	
5	3.9062	0.025694	2.2422	-1.8971	0.029608	0.90948	
6	-2.1914	-1.1872	-0.34067	4.1582	-1.2648	0.2317	
7	2.4957	-0.34693	-3.0857	-2.7598	2.4192	0.019633	
8	2.0353	3.1571	-1.8488	-2.4439	3.6219	0.00622	
9	-3.571	-0.75424	1.3114	-2.6853	-5.3414	-0.22336	

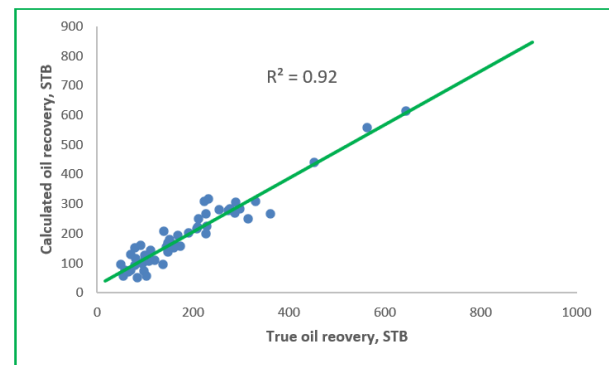


Figure 8: Cross plot of ANN oil recovery model of SSDR

3.6 Development of oil recovery for SWDR using ANN

The collected data sets are divided into three sets before building the ANN model to predict the oil recovery. 70% of the data sets are employed for the process of model training, while the remaining 30% of data sets are used for validation, and testing the performance of the model. Besides, all the data are normalized. In this work, an ANN model is developed to estimate the oil recovery as a function of the original oil

in place, porosity, water saturation, permeability, Pi/Pa ratio, net pay thickness, and, API. The model is built based on three layers. The first layer is the input layer, which has 7 neurons for inputs. 6 neurons contribute to the hidden layer, which is the second layer. The output layer, which has one neuron to predict the output parameter, oil recovery, is the third layer. we first examined the Tan-sigmoid as a transfer function at different numbers of hidden neurons (5, 6, 7, 8, 9, and 10) as presented in **Table 7**. We found the highest coefficient of determination and the lowest RMSE at $n = 6$. Then we examined the Tan-sigmoid as a transfer function at numbers of neurons of (5, 6, 7, 8, 9, and 10) as presented in **Table 8**. We found that the optimum transfer function is Logistic-sigmoid with 6 neurons where the coefficient of determination is 0.6 and MAE is 47.2 % To reach this fact a Levenberg-Marquardt technique was selected as a training algorithm and the pure-linear was examined to be the output function. **Figure 9** shows the layout of the proposed model and the characteristics of the proposed model are presented in **Table 9**.

Table 7: Oil recovery model accuracy at various numbers of neurons in the hidden layer with a tan-sigmoid transfer function for sandstone water drive:

	5	6	7	8	9	10
R2	0.52	0.55	0.26	0.59	0.59	0.6
SD	208.19	204.36	196.56	200.33	201.09	197.17
RMSE	185.27	177.74	221.49	172.76	172.18	173.53
RE	-26.66	-26.81	-20.26	-25.86	-26.46	-23.72
AE	51.88	48.57	51.37	47.36	48.55	50.93

Table 8: Oil recovery model accuracy at various numbers of neurons in the hidden layer with a log-sigmoid transfer function for sandstone water drive:

	5	6	7	8	9	10
R2	0.58	0.94	0.57	0.57	0.5	-0.11
SD	206.84	194.4	214.91	200.3	211.93	198.81
RMSE	172.82	169.98	174.38	174.33	182.63	210.74
RE	-25.58	-25	-31.69	-26.05	-29.2	-33.43
AE	47.25	47.22	49.12	47.73	51.05	56.85

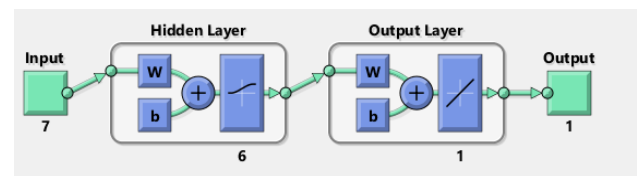


Figure 9: Layout of the proposed ANN oil r

Table 9: Characteristics of ANN oil recovery model of sandstone water drive recovery model for sandstone water drive

Parameter	Value
ANN layers number	3
neurons number in the input layer	7
optimum neuron number in the hidden layer	6
training algorithm for the neural network	Levenberg-Marquardt
transfer function of the hidden layer	Log sigmoid
transfer function of the output layer	pure linear

The developed model for oil recovery using the ANN can be expressed as follows:

First, the input parameters are normalized using the following expressions:

$$\begin{aligned} N_n &= 0.001099 * N \\ &- 1.265934 \end{aligned} \quad (21)$$

$$\begin{aligned} \phi_n &= 8.333333 * \phi \\ &- 1.916667 \end{aligned} \quad (22)$$

$$\begin{aligned} (S_w)_n &= 4.347826 * S_w \\ &- 1.304348 \end{aligned} \quad (23)$$

$$\begin{aligned} k_n &= 0.000401 * k \\ &- 1.002804 \end{aligned} \quad (24)$$

$$\begin{aligned} (P_i/P_a)_n &= 0.143375 * (P_i/P_a) \\ &- 1.143375 \end{aligned} \quad (25)$$

$$\begin{aligned} h_n &= 0.003339 * h \\ &- 1.003339 \end{aligned} \quad (26)$$

$$\begin{aligned} API_n &= 0.033898 * API \\ &- 1.033898 \end{aligned} \quad (27)$$

For $i = 1$ to no of neurons and for $j = 1$ to no of inputs, the inputs for the hidden are calculated from **Eq. 18**. The oil recovery can be calculated by the following expression:

$$\begin{aligned} REC &= 734.5 \left[-0.0228 \right. \\ &\quad \left. + \sum_{i=1}^n W_{i,j} \left(\frac{1}{1 + \exp(-S_i)} \right) \right] \\ &\quad + 735.5 \end{aligned} \quad (28)$$

The proposed oil recovery model's coefficients are listed in **Table 10**. The regression plot for the oil recovery model of SSDR, as indicated in **Figure 10** presents the relation between the calculated and the true oil recovery for all data sets. The fit is accepted where the coefficient of determination is 0.94.

Table 10: Weights and Biases of the Oil Recovery ANN Model for SWDR

Neuron	W _{i,1}	W _{i,2}	W _{i,3}	W _{i,4}	W _{i,5}	W _{i,6}	W _{i,7}	W _{hi}	bi
1	3.0197	0.3794	0.00839	-0.8997	1.9351	-0.641	0.45729	-3.0901	1.4917
2	4.899	3.0177	0.13528	-1.3855	-0.2651	-2.413	2.6506	-2.267	0.60098
3	-1.2311	1.4232	3.8264	1.0633	2.5017	-2.2841	-2.7343	3.5377	-1.2004
4	-0.97186	2.1979	0.71725	2.9217	-2.1781	-1.0218	-3.7718	1.0167	0.51583
5	-3.8	-5.905	-0.1087	0.51892	2.473	2.1984	-1.4769	4.2591	-0.1458
6	-0.98269	1.7365	0.77143	2.1174	-0.7372	2.1245	-0.5136	-3.7223	0.27164

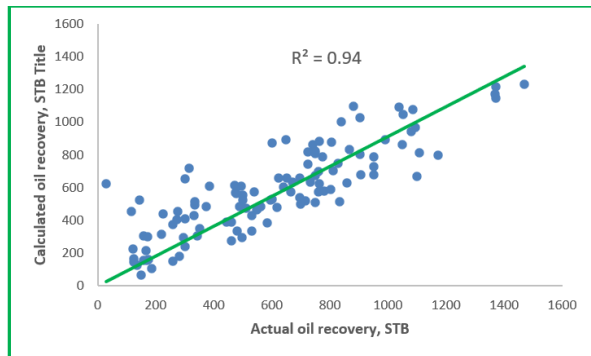


Figure 10: Cross plot of ANN oil recovery model of SWDR

4. Conclusions

From the work, the following conclusions may be drawn:

1. A correlation was developed (139 data sets) in this study to predict the oil recovery for sandstone solution gas drive reservoirs (SGDR) using NLMR with a coefficient of determination, R^2 of 0.90 (compared to 0.87 for Gulstad correlation).
2. A new ANN model was developed to calculate the oil recovery of sandstone solution gas drive reservoirs giving an R^2 of 0.92.

3. A correlation was developed (111 data sets) in this study to predict the oil recovery for sandstone water drive reservoirs (SWDR) using NLMR with a coefficient of determination, R^2 of 0.92 (compared to 0.91 for American Petroleum Institute, API correlation).
4. A new ANN model was developed to calculate the oil recovery of sandstone water drive reservoirs giving an R^2 of 0.94.

Nomenclature

B_{oa} : the oil formation volume factor at the abandonment pressures, bbl/STB.

B_{oi} : the oil formation volume factor at the initial pressure, bbl/STB.

h: net pay thickness

k: permeability, md.

N : the original oil in place, OOIP, STB

N_{cal} : the calculated original oil in place, STB

P_a : the abandonment pressure, psi.

P_i : the initial pressure, psi.

NT STOIIP: the stock-tank oil initially in place (STB/NAF)

STOOIP: Stock tank original oil in place.

S_{wi} : the initial water saturation, dimensionless.

T: Reservoir temperature, °F

ϕ : porosity, dimensionless

μ_{oa} : the oil viscosity at the abandonment pressure, cp.

μ_{ob} : the oil viscosity at the bubble point pressure, cp.

μ_{oi} : the oil viscosity at the initial pressure, cp.

μ_{wi} : the water viscosity at the initial pressure, cp.

G: Net-to-gross ratio

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